Probabilistic Assessment of Overtopping Reliability of a Dam

A.Melih Yanmaz, M.Engin Gunindi
Department of Civil Engineering, Middle East Technical University
Ankara, TURKEY

Abstract

Probabilistic methods, which consider resistance and load parameters as random variables, are more realistic than conventional deterministic methods for determining the safety level of a dam. This study is based on a probabilistic assessment of overtopping reliability of a flood detention dam. In the analysis, the inflow hydrograph parameters and the initial reservoir level prior to the allocation of the flood in the reservoir are accepted as random variables. A bivariate flood frequency analysis is performed in which the annual maximum peak discharges and the surface runoff volumes of the floods are handled as the random variables, using bivariate gamma probability density function. This operation yielded a number of flood hydrographs having different peak discharges and runoff volumes under a particular return period. Using this information, family of return period curves relating the runoff volume to peak discharge are generated at the dam site. Maximum reservoir elevation is determined by performing reservoir routing based on Monte Carlo simulation. This calculation is repeated for various combinations of possible flood hydrographs under a constant return period to observe the variation of overtopping reliability. It is, therefore, intended to find the most critical case that is likely to occur at the dam site. The findings of the present analysis may be used in decision-making for the crest elevation of the dam, which is safe against overtopping.

Keywords: flood, overtopping, dam, probabilistic assessment, reliability, Monte Carlo simulation

Introductory Remarks

Probabilistic design approaches enable assessment of various reliability levels under different combinations of random design parameters. Using relevant data of sufficient size and precision, probability-based safety levels of existing dams may be evaluated against various tendencies. This manipulation is of importance since the majority of the existing dams were designed and constructed during the last century using conventional design procedures, which were normally deterministic in nature. Therefore, the adequacy of these dams needs to be checked with respect to the current safety and conformity. Such an analysis gains importance especially if pronounced variations occur in many aspects, e.g. aging and deterioration of constructional materials, deficiencies resulting from various structural, hydraulic, and geotechnical aspects, changes in watershed characteristics, alteration of operational policies, inability of old design methods to consider random nature of loading and resistance parameters, etc. New dams need also to be designed using contemporary techniques. Probabilistic design and operation approaches are superior to the deterministic approaches as they provide more realistic information for site-specific conditions.

The aim of this study is to present a methodology for computing the overtopping reliability of a flood detention dam using a bivariate flood frequency analysis. It is also intended to compare the results obtained from conventional approaches with those of the contemporary techniques. To this end, the results of univariate and bivariate flood frequency analyses are compared with each other. Furthermore, the findings of deterministic reservoir routing are also compared with the probabilistic routing procedure, which is based on Monte Carlo simulation. An earth-fill dam is planned to accommodate and attenuate frequent floods in Fol Creek Basin, located in Eastern Black Sea region of Turkey. Overtopping occurs when the reservoir surface elevation exceeds the dam crest elevation during severe floods or severe wave action, which may be induced by wind, landslides, earthquakes, etc. In this study, the adequacy of the dam is assessed only with respect to the overtopping probability of the crest against flood action. The other possible failure modes, such as sliding of slopes, excessive seepage, earthquake, etc., will not be taken into account. Assessment of the overtopping possibility using such a probabilistic approach is superior to the deterministic approach in which the safety level is assessed only by checking whether or not SM>0 or SF>1.0, where SM and SF are the safety margin and safety factor, respectively. These parameters are expressed as SM= h_c-h_m, and SF=h_c/h_m, where h_c and h_m are the elevations of the dam crest and the maximum reservoir level, respectively. Since possible variations of design variables are considered in a probabilistic analysis, it may be possible to obtain intolerably high probability of failure. However, in case of a deterministic analysis using the same mean design values as the probabilistic approach, the requirements for SM>0 and SF>1.0 may be satisfied. Therefore, computation of probability of failure or its complement, reliability, would provide more realistic information than deterministic approach (Yanmaz and Gunindi, 2004).

Multivariate Flood Frequency Analysis

Conventional flood frequency analysis is carried out using annual flood peak discharges to obtain extreme flood peaks using the statistical properties of the sample data. However, such applications are normally incapable of giving adequate information for the floods since the whole event would be modelled more correctly by the joint consideration of flood peaks, volumes, and durations. There is a growing research activity on multivariate flood frequency analysis. Preliminary studies on the theoretical establishments and applications have been carried out by Sackl and Bergmann (1987), Goel et al. (1998), Escalante-Sandoval and Raynal-Villaseñor (1998), Yue (2000), Yue (2001), Yue et al. (2001), Yue (2002), Yue and Rasmussen (2002), Michele et al. (2005), etc. Hydrologic events having skewed distributions, such as flood peak discharge and flood volume may follow gamma type probability distribution. Smith et al. (1982) established the theoretical basis of five-parameter bivariate gamma distribution. Yue (2001) and Yue et al. (2001) applied a bivariate gamma distribution for investigating the joint probability behavior of these events. The joint probability density function (PDF) and cumulative density function (CDF) of the five-parameter bivariate gamma distribution with variates X and Y are as follows (Smith et al., 1982; Yue, 2001):

$$f(x,y)=\begin{cases}\dfrac{K_1}{K_2}\sum\limits_{k=0}^{\infty}\sum\limits_{j=0}^{\infty}c_{jk}\left(\beta_x x\right)^{j}\left(\eta\beta_y y\right)^{j+k} & \text{if } \rho>0\\ f_x(x).f_y(y) & \text{if } \rho=0\end{cases} \tag{1}$$

$$F(x,y)=\begin{cases}J\sum\limits_{k=0}^{\infty}\sum\limits_{j=0}^{\infty}d_{kj}H\left(\gamma_x+j,\dfrac{\beta_x x}{1-\eta}\right)H\left(\gamma_y+j+k,\dfrac{\beta_y y}{1-\eta}\right) & \text{if } \rho>0\\ F(x).F(y) & \text{if } \rho=0\end{cases} \tag{2}$$

in which $x,y\geq 0,\ 0<\eta<1,\ \gamma_y\geq\gamma_x$, and $0\leq\rho<\eta\sqrt{\gamma_x/\gamma_y}$; and

$$K_1=\left(\beta_x x\right)^{\gamma_x-1}\left(\beta_y y\right)^{\gamma_y-1}\exp\left(-\frac{\beta_x x+\beta_y y}{1-\eta}\right) \tag{3}$$

$$K_2=\left(1-\eta\right)^{\gamma_x-1}\,\Gamma\left(\gamma_x\right)\Gamma\left(\gamma_y-\gamma_x\right) \tag{4}$$

$$c_{kj}=\frac{\Gamma\left(\gamma_y-\gamma_x+k\right)}{\left(1-\eta\right)^{2j+k}\,\Gamma\left(\gamma_y+j+k\right)j!\,k!} \tag{5}$$

$$\eta=\rho\sqrt{\frac{\gamma_y}{\gamma_x}} \tag{6}$$

$$J=\frac{\left(1-\eta\right)^{\gamma_y}}{\Gamma\left(\gamma_x\right)\Gamma\left(\gamma_y-\gamma_x\right)} \tag{7}$$

$$d_{kj}=\frac{\eta^{j+k}\,\Gamma\left(\gamma_y-\gamma_x+k\right)}{\Gamma\left(\gamma_y+j+k\right)j!\,k!} \tag{8}$$

$$H\left(a,z\right)=\int_0^z t^{a-1}e^{-t}dt \tag{9}$$

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt \tag{10}$$

$$\rho = \frac{E\left((X - M_x)(Y - M_y)\right)}{S_x S_y} \tag{11}$$

where γ and β are the scale and shape parameters of the marginal gamma distributions, respectively, $\gamma = M^2/S^2$, $\beta = M/S^2$, M and S are the mean and standard deviation of the sample data sets using the method of moments, respectively. In Eq. (6), η is defined as the association parameter between X and Y, ρ is the product-moment correlation coefficient of X and Y estimated from Eq. (11). The PDFs $f_X(x)$ and $f_Y(y)$, of the marginal distributions of X and Y are computed from (Yue, 2001)

$$f_X(x) = \frac{1}{\Gamma(\gamma_x)} x^{\gamma_x - 1} \beta_x^{\gamma_x} e^{-\beta_x x} \tag{12}$$

$$f_Y(y) = \frac{1}{\Gamma(\gamma_y)} x^{\gamma_y - 1} \beta_y^{\gamma_y} e^{-\beta_y y} \tag{13}$$

Since Eqs. (12) and (13) cannot be solved analytically; CDFs of X and Y are computed by the numerical integration of these equations (Yue, 2001). Application of this methodology is illustrated in the following section.

Case Study

In the case study, Fol Creek Basin in Eastern Black Sea region of Turkey has been selected. This basin is subject to frequent floods leading to loss of several lives and considerable damage to all types of facilities. It is an elongated basin having an area of 219.6 km^2. There is a recording stream-gauging station close to the outlet. The main branch is a fourth-order stream on the basis of Horton's classification of stream ordering. It flows almost in a V-shaped valley with steep side slopes. The maximum basin relief is 2340 m. The overall runoff coefficient in the basin is quite large due to the steep slopes, mainly clayey and rocky formations (Yanmaz and Coskun, 1995). Town of Vakfikebir is located at the outlet of the basin, which is adversely affected by the frequent floods since various types of structures and establishments are placed on the floodplains. Following the heavy flood in June 1990, this town has been seriously inundated and the highway on the left bank of the Fol Creek has been collapsed (Yanmaz and Bilen, 2000). Upon repairing this highway, partial damages have been observed during the successive floods. Therefore, intensive structural measures need to be taken into consideration. Yanmaz and Bilen (2000) carried out hydroeconomic analyses for determining the optimum return period of flood and optimum bank protection facilities. Considering local hydrometeorologic conditions, however, these structural measures need to be supplemented by an upstream flood detention dam to facilitate high degree of attenuation such that the downstream inundation level is reduced. Based on the inspection of local topographic and geologic conditions, construction of a flood detention dam at a suitable axis, 2511 m upstream of the Black Sea shoreline, is proposed. Suitable dimensions will be assigned to the structure and the adequacy of the dam against overtopping induced by flood action will be checked.

Flood Frequency Analyses

A preliminary analysis is carried out to observe the effect of flood frequency analysis on the design. The annual series of peak discharges, which were recorded by the stream-gauging station, are presented in Table 1 together with the corresponding direct runoff volumes. Yanmaz and Bilen (2000) carried out a flood frequency analysis for the annual maximum flows of the Fol Creek basin using a computer program developed by Hosking (http://lib.stat.cmu.edu/general/lmoment). The program determines the magnitude of discharges corresponding to specified return periods for various probability distributions using the theory of L-moments for parameter estimation. It is applicable to either regional or at-site frequency analysis. Since there is only one stream-gauging station in the Fol Creek basin, the at-site option of the program was run. The results of the analysis are presented in Figure 1, in which 12 different probability distributions (Gamma (G), Generalized Extreme Value (GEV), Generalized Normal (GN), Extreme Value Type 1 (EV1), Normal (N), Pearson Type III (PE3), Generalized Pareto (GPA), Generalized Logistic (GL), Kappa (KAP), Wakeby (WAK), 2-Parameter Log-normal (LN2), and log-Pearson Type III (LPT3)) were used. As can be seen from Figure 1, the discrepancy among the data points increases with increasing return period.

Table 1. Annual Maximum Flood Events of the Fol Creek

Water Year	Q_p (m^3/s)	V (10^6 m^3)
1972	35.30	3.92
1976	31.00	1.50
1979	14.80	2.39
1980	73.50	3.19
1983	177.00	8.02
1984	41.15	3.82
1985	35.40	2.21
1986	45.00	2.23
1987	72.50	2.66
1988	52.80	3.27
1989	55.20	4.17
1993	41.79	2.10
1995	102.09	3.90
1996	101.96	2.54
1999	71.60	1.79
2000	196.72	3.21
2001	31.66	1.78
2002	74.08	1.48
2003	65.52	5.66
2004	71.12	4.10

Bivariate flood frequency analysis is carried out using gamma marginals for the flood peak discharges and direct runoff volumes. To examine the goodness of fit of the gamma distribution to the flood peak discharges, Q_p, and volumes, V, the Kolmogorov–Smirnov (KS) test is applied. The critical value of the KS test is 0.290 at the significance level of 0.05. The KS test statistics are 0.198 for the flood peak discharge, and 0.204 for the flood volume. Thus, the null hypothesis H_0 that the underlying distributions of these flood characteristics are the gamma distribution type is accepted at a significance level of 0.05. The CDF of annual flood events following bivariate gamma distribution is obtained using Eqs. (1) through (11). To obtain a relation between Q_p and V, successive values of CDF are considered and the corresponding values of these variables are found by trail and error. As a result, sets of equal return period curves correlating Q_p-V pairs are obtained as shown in Figure 2 in which T_r is the return period. Although these curves extend asymptotically along the axes, they should be limited by proper upper and lower bounds since very large values of these variables have no physical significance. To this end, the criterion suggested by Hable (2001) is used in which the aforementioned curves are bounded by lines passing through the origin having slopes of maximum $r_{max}=V/Q_p$ and minimum $r_{min}=V/Q_p$ among the data points (Fig. 2). Since the available record length is limited by 20 years, forecasts for very big return periods may be subject to high uncertainty. That is why this study will only deal with a return period of 100 years. To examine the effect of Q_p and V pairs under 100 years of return period, five cases are considered, which are designated by letters A through E together with their corresponding characteristic values (See Fig. 2). The average flood peak values of the univariate flood frequency analysis are compared with the peak discharge ranges obtained from the bivariate analysis in Table 2. As can be seen from this table, bivariate analysis yielded a wide range of peak discharges within the upper and lower bounds for the return periods considered. Another by-product of this analysis is that the average peak discharges obtained from the univariate analysis are smaller than the lower limit of the bivariate case. Therefore, the design performed using univariate analysis would lead to underestimates compared to the case of bivariate analysis.

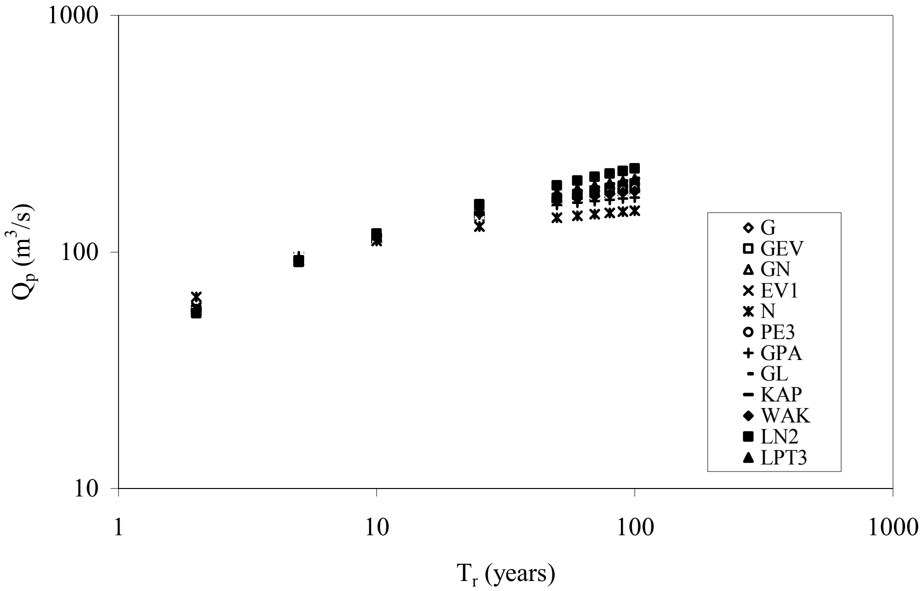

Figure 1. Univariate flood frequency analysis for the case study

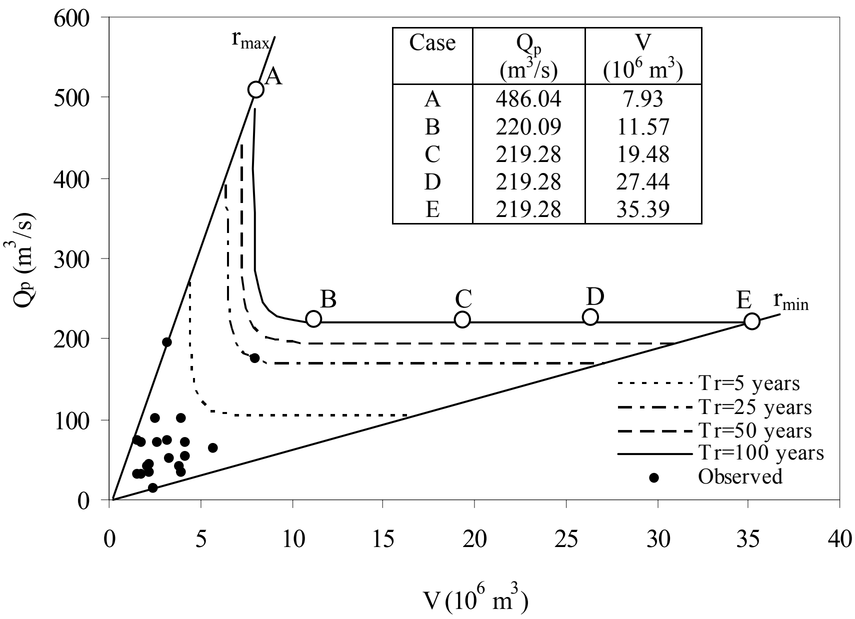

Figure 2. Correlation between Q_p and V for various return periods

Table 2. Comparison of Q_p values

T_r	Q_p (m^3/s) (univariate)	Q_p (m^3/s) (bivariate)
2	57.5	60-181.4
5	92.4	102.9-269.0
10	115.6	131.9-325.3
25	144.7	167.9-391.9
50	166.2	193.8-440.3
100	187.7	219.6-486.0

The next step is the generation of hydrographs corresponding to the aforementioned cases such that the worst possible case, giving the highest probability of overtopping, would be determined. The respective hydrographs are obtained using a technique proposed by Aldama and Ramirez (1999), which is based on Hermetian polynomials. Hydrograph coordinates for this method are obtained from

$$Q(t;Q_p,t_p,V)=\begin{cases} Q_p\left[3(t/t_p)^2-2(t/t_p)^3\right] & t\in\left[0,t_p\right] \\ Q_p\left[1-3(t/t_p)^2/(2VQ_p^{-1}-t_p)^2+2(t/t_p)^3/(2VQ_p^{-1}-t_p)^3\right] & t\in\left[t_p,t_b\right] \\ 0 & t\in(-\infty,0)\cup(t_b,\infty) \end{cases} \quad (14)$$

where t_p is the time to peak ($2V/3Q_p$) and t_b is the base time ($3t_p$) (Chow, 1964). The resulting hydrographs are presented in Figure. 3.

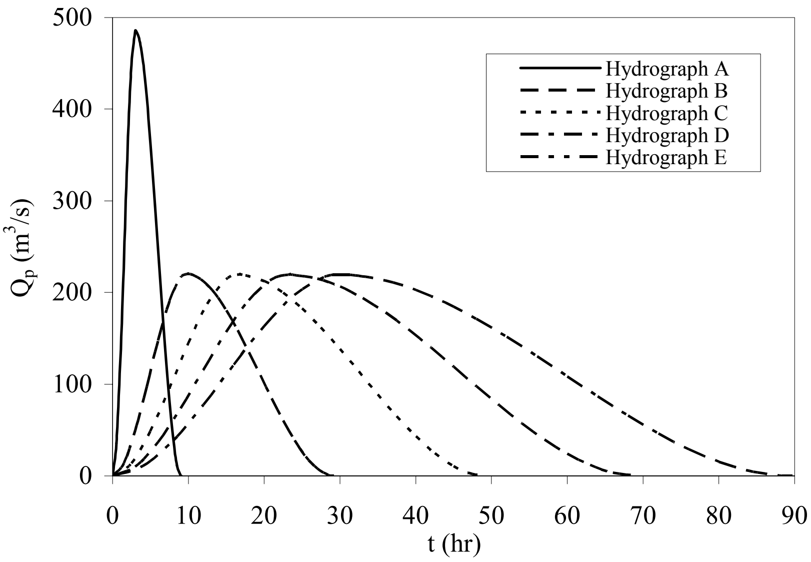

Figure 3. Hydrographs for cases A through E

The cases A and E represent two extreme conditions within the limits of upper and lower bounds of Q_p and V pairs under the return period of 100 years. Hydrograph A has a high peak discharge with a relatively short base time and small volume, whereas hydrograph E has a peak discharge, which is less than the half of that of the hydrograph A but has a relatively large flood volume and base time. Therefore, concerning the storage ability of the reservoir, hydrograph E is expected to yield the worst conditions against overtopping possibility and to release high outflows during a long duration, compared to the other possible hydrographs. Furthermore, the duration of downstream inundation increases as the outflow experiences the cases from A to E.

Computation of Overtopping Reliability

The overtopping reliability of the flood detention dam is accepted to be the probability of the dam crest elevation being equal to or exceeding the maximum reservoir level expected to occur during the passage of floods. It is clear that different maximum reservoir levels would attain during the passage of the aforementioned inflow hydrographs. The maximum reservoir levels that may occur under the hydrographs A through E, can be obtained from a reservoir routing. The routing equation is

$$\frac{dh}{dt}=\frac{I(t)-Q(h)}{A(h)} \quad (15)$$

where h is the stage measured from the thalweg, t is the time, I(t) is the inflow hydrograph, Q(h) is the outflow, and A(h) is the area-elevation relation of the reservoir. To reduce the number of possible design configurations with respect to dimensions of various appurtenances of the dam and hydrologic parameters, the case study will be carried out by considering only the following features. In this study, a single bottom outlet of 1 m diameter is planned. The crest elevation of the riser inlet is taken as 3 m above the thalweg elevation. An uncontrolled overflow spillway is planned to have a crest length of 40 m having a 1 m thick pier in the middle. The outflow-elevation relation is obtained from the energy equation between the upstream and downstream until the spillway crest elevation. The minor losses for the entrance, trashrack, bend, and exit are considered as well as the frictional losses along the bottom outlet. For the elevations above the spillway crest, the aforementioned outflow is incremented by the spillway discharge equation. The outflow values are obtained for incremental values of stage and are converted to best-fit equations for numerical routing computations. The area-elevation relation of the reservoir is obtained from the respective maps as

$$A(h) = -4.02 * 10^{-6} h^6 + 0.00165h^5 - 0.257h^4 + 20.92h^3 - 895.2h^2 + 35200h \tag{16}$$

where h is the stage in m and A(h) is the corresponding surface area in m^2. In the case study, a 30 m high earth-fill dam is considered to check the adequacy of the reservoir for accommodating the flood volumes under 100 years of return period. The dam has an overflow spillway with a crest height of 25 m. The initial water depth in the reservoir is in fact a random variable, which depends on the reservoir operation and management as well as the dimensions of the outlet facilities. Since this is a new dam to be constructed, no statistical information is available for the temporal variation of mean operating reservoir levels. That is why an arbitrary value of 3.2 m is assigned to the initial water depth in the reservoir. When the inflow, outflow, and the area-elevation relations are inserted in Eq. (15), a first order nonlinear differential equation is obtained, which can be solved numerically using Euler's technique. In the routing computations, the stage is accepted as a random variable whose incremental values are obtained from the Monte Carlo simulations based on the solution of Eq. (15). The uncertainty associated with the random stage can be expressed in terms of its coefficient of variation. However, the validity of the results of an uncertainty analysis is dependent on the correct choice of the coefficient of variation and probability distribution of the variables involved in the phenomenon (Yanmaz, 2003). With reference to Johnson (1996), Yanmaz (2000), and Yanmaz and Cicekdag (2001), a coefficient of variation of 0.05 is taken for the stage and a normal probability density function is assigned. In the Monte Carlo analysis, the number of simulation cycles influences the level of reliability. The number of cycles must be large in order to obtain a significant sampling of simulation events such that the results are close to the exact values. The accuracy of the mean reliability under a particular simulation cycle may be estimated by the coefficient of variation of reliability, Λ_r, which decreases with increasing sample size. Therefore, simulations should be carried out several times for large cycles such that the corresponding value of Λ_r is almost constant and relatively small (Yanmaz, 2003). Such an analysis is carried out to obtain 20,000 cycles for the determination of successive stage values in the routing computation using Monte Carlo simulation (See Fig. 4).

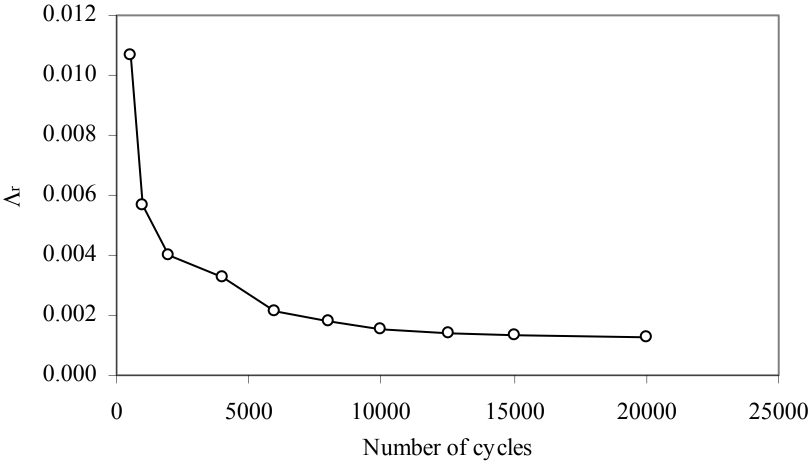

Figure 4. Variation of coefficient of variation of reliability against number of cycles

The resultant stage hydrographs for the cases A through E are then obtained from the solution of Eq. (15) (See Fig. 5). Although the hydrograph A has the greatest peak discharge, it leads to the smallest maximum stage compared to the other hydrographs since it has the smallest flood volume and base time. On the contrary, the hydrograph E yields high stages for a considerably long duration because of its large volume (Fig.5). The reliability of the crest overtopping is obtained using Monte Carlo simulations in which random numbers are generated for P(SM>0), where P is the probability. To observe the effect of probabilistic routing, Eq. (15) is also solved for the deterministic case i.e. h is not treated as a random variable. The stage hydrographs are obtained for the cases B and D and compared with the probabilistic approach in Figure 6, which reveals that deterministic stages are slightly greater than those of probabilistic stages. This result clearly shows that type of the routing procedure, i.e. deterministic or probabilistic, influences the level of reliability. Variation of the deterministic safety factor against dimensionless storage, $V^*=V/V_d$, where V_d is the volume of the dam body, is shown in Figure 7. Similar information is also provided in Figure 8 for the probabilistic approach in which the variation of overtopping reliability is shown with respect to the dimensionless storage. This study result that deterministic safety factors are greater than unity and overtopping reliabilities are relatively high for the cases tested. Therefore, the dam having the proposed dimensions is accepted to be adequate against flood accommodation.

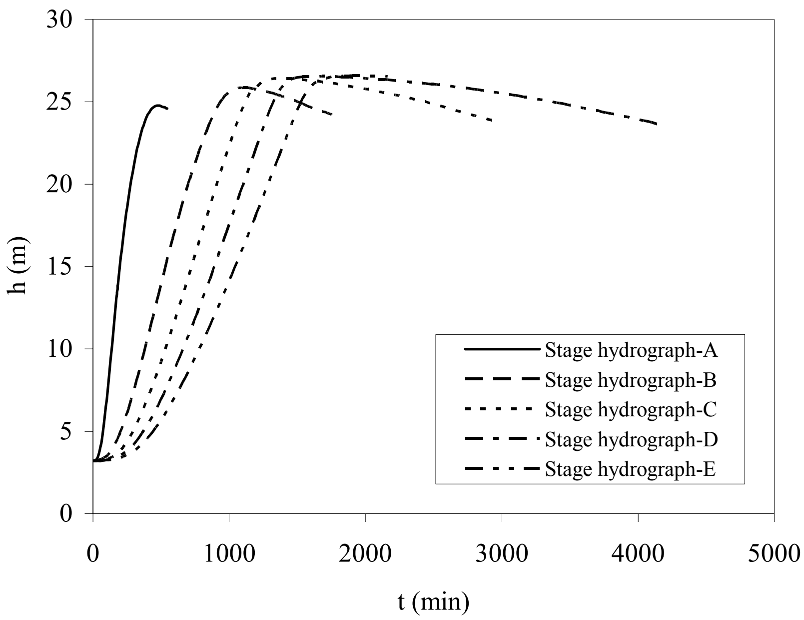

Figure 5. Stage hydrographs

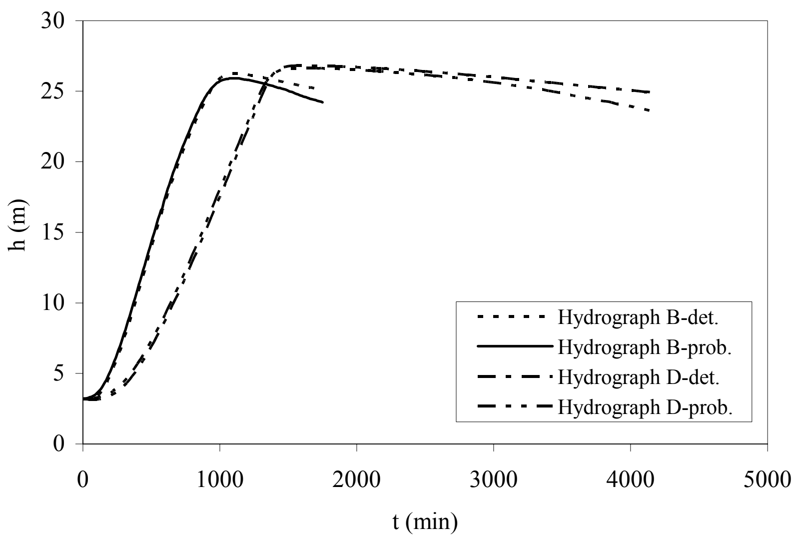

Figure 6. Comparison of deterministic and probabilistic stage hydrographs

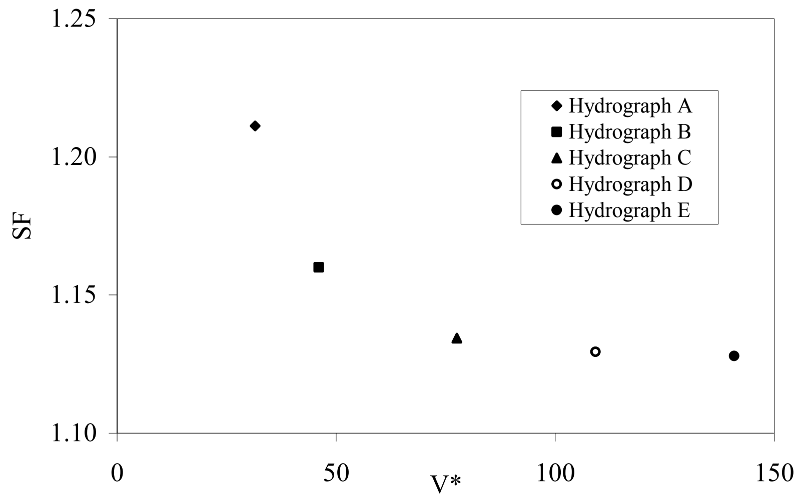

Figure 7. Variation of the safety factor with the dimensionless storage

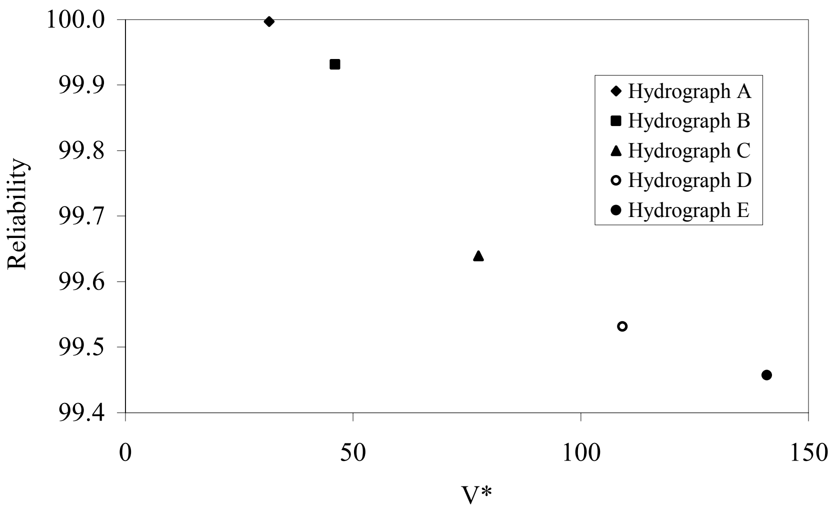

Figure 8. Variation of the reliability with the dimensionless storage

Conclusions

Overtopping reliability of an earth-fill flood detention dam, 30 m high, is investigated. The design flood peak discharge and flood volume are determined using a bivariate gamma distribution with five parameters. Equal return period curves relating these variables are generated. Using this information, a number of hydrographs having different characteristics under a return period of 100 years are obtained using the technique proposed by Aldama and Ramirez to observe their effects on the overtopping reliability of the dam. An overflow spillway 25 m high and 40 m long is proposed. Flood frequency analyses are performed for univariate and bivariate cases. The average peak discharges of different distributions in univariate analysis are observed to be smaller than the lower limits of the peak discharge ranges of the bivariate analysis for all return periods. This result implies that the deterministic approach yield underestimates in the design compared to the probabilistic approach. Various possible hydrographs tested under a 100-year return period yielded relatively high values of overtopping reliability. Therefore, implementation of the flood detention dam with the proposed dimensions is assessed to be feasible from viewpoint of flood attenuation. The effects of initial water level in the reservoir, the coefficient of variation and the PDF assigned to water stage, and various relevant dimensions of the dam on the overtopping reliability may also be examined in a future study.

References

Aldama, A. A., A.I. Ramirez, 1999: *A new approach for dam design flood estimation. IAHR Congress Proceedings, Graz, Austria.*

Chow, V. T., 1964: *Handbook of Applied Hydrology, McGraw-Hill, New York.*

Escalante-Sandoval, C.A., J. A. Raynal Villaseñor, 1998: *Multivariate estimation of floods: the trivariate gumbel distribution. J. Statist. Comput. Simul., OPA N.V., India, 61, 313-340.*

Goel, N. K. S. M. Seth, S. Chandra, 1998: *Multivariate modeling of flood flows." J. Hydraul. Eng. 124(2), 146-155.*

Hable, O., 2001: *Multidimensional probabilistic design concept for the estimation of the overtopping probability of dams. Ph.D. thesis, Technical University of Graz.*

Johnson, P.A., 1996: *Uncertainty of hydraulic parameters, J. Hydraul. Eng., 122(2), 112-114.*

Michele, C.De, G. Salvadori, M. Canossi, A. Petaccia, R. Rosso, 2005: *Bivariate statistical approach to check adequacy of dam spillway, J. Hydrologic Eng., 10(1), 50-57.*

Sackl, B. H., Bergmann, 1987: *A bivariate flood model and its application Hydrologic Frequency Modeling. V. P. Singh (ed.), D. Reidel Pub. Company, The Netherlands, 571-582.*

Smith, O. E., S.I. Adelfang J.D., Tubbs, 1982: *A Bivariate gamma probability distribution with application to gust modeling, NASA Technical Memorandum NASA TM-82483, Alabama.*

Yanmaz, A. M., F.Coşkun,1995: *Hydrological aspects of bridge design: case study, J. Irrig. and Drain. Eng., 121(6), 1-8.*

Yanmaz, A. M., S. Bilen, 2000: *Selection of optimum riverbank stabilization facility, Proc. 4th Int. Conf. on Hydroscience and Engineering, Seoul, Korea.*

Yanmaz, A.M., 2000: *Overtopping risk assessment in river diversion facility design, Canadian J. Civil Eng., 27, 319-326.*

Yanmaz, A.M., O. Cicekdag, 2001: *Composite Reliability Model for Local Scour Around Cylindrical Bridge Piers, Canadian J. Civil Eng., 28(3), 520-535.*

Yanmaz, A.M., 2003: *Reliability-based assessment of erodible channel capacity, Turkish J. of Eng. and Env. Sciences, 27(4), 265-273.*

Yanmaz, A.M., E. Gunindi, 2004: *Probability-based procedures and risks in dam design, Proc.1st National Symposium on Dams and Hydroelectric Power Plants", State Hydraulics Works, Ankara, Turkey (in Turkish).*

Yue, S., 2000: *The bivariate lognormal distribution to model a multivariate flood episode. Hydrological Processes. John Wiley and Sons Ltd., 14, 2575-2588.*

Yue, S., 2001*: A bivariate gamma distribution for use in multivariate flood frequency analysis. Hydrological Processes. John Wiley and Sons Ltd., 15, 1033-1045.*

Yue, S., T.B.M.J. Ouarda, B. Bobée, 2001: *A review of bivariate gamma distributions for hydrological application." J. Hydrology, Elsevier Science Ltd., 246, 1-18.*

Yue, S., P. Rasmussen, 2002: *Bivariate frequency analysis: discussion of some useful concepts in hydrological application. Hydrological Processes. John Wiley and Sons Ltd., 16, 2881-2898.*

Yue, S., 2002: *The bivariate lognormal distribution for describing joint statistical properties of a multivariate storm event. Environmetrics. John Wiley and Sons Ltd., 13, 811-819.*